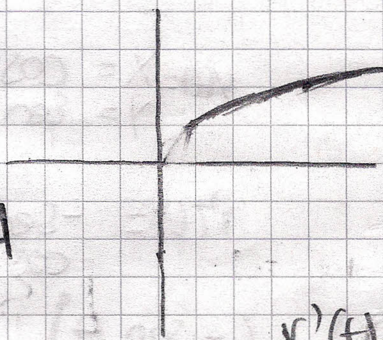
TALLER 11

(1) Evalue las siguientes integrales de linea donde C es la curva dad

(a) $\int_C (x^2y^3-\sqrt{x})\,dy$ $\qquad C = y = \sqrt{x}$ de $(1,1)$ a $(4,2)$



$\begin{cases} x = t \\ y = \sqrt{t} \end{cases} \qquad 1 \le t \le 4$

$r'(t) = \begin{cases} 1 \\ 1/2\sqrt{t} \end{cases}$

$$\int_1^4 \begin{pmatrix} 0 \\ t^2\, t\sqrt{t} - \sqrt{t} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1/2\sqrt{t} \end{pmatrix} dt$$

$$\cancel{\sqrt{t}}\,(t^3 - 1) \cdot \frac{1}{2\cancel{\sqrt{t}}}$$

$\frac{1}{2}\int_1^4 t^3 - 1 \, dt$ $\qquad \frac{1}{2}[60] - \frac{1}{2}\left[\frac{1}{4} - 1\right]$

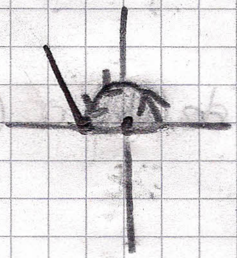
$\frac{1}{2}\left(\frac{t^4}{4} - t\right)\Big|_1^4$ $\qquad \frac{1}{2}\left(30 - \frac{1}{8} + \frac{1}{2} - 1\right)$

$\frac{1}{2}\left(\frac{4^4}{4} - 4\right)$ $\qquad 30 - \frac{61}{2} - \frac{1}{8}$

$$\boxed{\frac{243}{8}}$$

(b) $\int_C \operatorname{sen}(x)\,dx + \cos y\,dy$

$C =$ Mitad superior de $x^2+y^2=2$ de $\begin{pmatrix}1\\0\end{pmatrix}$ a $\begin{pmatrix}-1\\0\end{pmatrix}$



$r(t) = \begin{matrix} x = \cos\theta \\ y = \operatorname{sen}\theta \end{matrix} \qquad 0 \leq \theta \leq \pi$

$r'(t) = \begin{matrix} -\operatorname{sen}\theta \\ \cos\theta \end{matrix}$

$$\int_0^{\pi} \begin{pmatrix} \operatorname{sen}(\cos\theta) \\ \cos(\operatorname{sen}\theta) \end{pmatrix} \cdot \begin{pmatrix} -\operatorname{sen}\theta \\ \cos\theta \end{pmatrix} d\theta$$

$$\int_0^{\pi} -\operatorname{sen}(\cos\theta)\cdot\operatorname{sen}\theta\,d\theta + \int_0^{\pi} \cos(\operatorname{sen}\theta)\cdot\cos\theta\, d\theta$$

$u = \cos\theta \qquad\qquad u = \operatorname{sen}\theta$

$du = -\operatorname{sen}\theta \qquad\qquad du = \cos\theta$

$$\int_{-1}^{1} \operatorname{sen} u\, du + \int_0^0 \cos(u)\, du$$

$$-\cos u \Big|_{-1}^{1} + \operatorname{sen} u \Big|_0^0$$

$$-\cos(1) + \cos(-1)$$

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(c) $\int z\,dx + x\,dy + y\,dz$

C: $x = t^2$, $y = t^3$, $z = t^2$, $0 \le t \le 1$

$\begin{pmatrix} z \\ x \\ y \end{pmatrix} \begin{pmatrix} 2t \\ 3t^2 \\ 2t \end{pmatrix}$ $= \int_0^1 2t^3 + 3t^4 + 2t^4$

$\begin{pmatrix} t^2 \\ t^2 \\ t^3 \end{pmatrix} \begin{pmatrix} 2t \\ 3t^2 \\ 2t \end{pmatrix}$ $= \dfrac{2t^4}{4} + \dfrac{3}{5}t^5 + \dfrac{2}{5}t^5$

$\dfrac{1}{2} + \dfrac{3}{5} + \dfrac{2}{5}$ 1,2(

$\dfrac{1}{2} + \dfrac{1}{1}$ $\dfrac{1+2}{3}$ $= \boxed{\dfrac{3}{2}}$

t^2

(d) $\int_C x^2\,dx + y^2\,dy + z^2\,dz$

C= Esta formada por segmentos de recta de $\begin{pmatrix} 0 \\ 8 \end{pmatrix}$ a $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ y de $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$

Parametrizamos los dos segmentos

1er segmento

$x = 0 + (t)(1)$
$y = 0 + (t)(2)$
$z = 0 + (t)(-1)$

$\boxed{\begin{matrix} x = t \\ y = 2t \\ z = -t \end{matrix}}$

$r'(t) = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ $0 \le t \le 1$

2do segmento

$x = 1 + (t)(2)$
$y = 2 + (t)(0)$
$z = -1 + (t)(+1)$

$\boxed{\begin{matrix} x = 1+2t \\ y = 2 \\ z = -1+t \end{matrix}}$

$r'(t) = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ $0 \le t \le 1$

$(t-1)$

Planteamos la I de línea

$$\int_0^1 \begin{pmatrix} x^2 \\ y^2 \\ z^2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} dt + \int_0^1 \begin{pmatrix} x^2 \\ y^2 \\ z^2 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} dt$$

$$\int_0^1 \begin{pmatrix} t^2 \\ 4t^2 \\ t^2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} dt + \int_0^1 \begin{pmatrix} 1+4t+4t^2 \\ 4 \\ t^2-2t+1 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} dt$$

$$\int_0^1 \cancel{t^2} + 8t^2 - \cancel{t^2}\, dt + \int_0^1 2 + \cancel{8t} + 8t^2 + t^2 - \cancel{2t} + 1$$

$$\int_0^1 8t^2\, dt + \int_0^1 9t^2 + 6t + 3$$

$$\frac{8}{3} + \frac{9}{3}$$

$$\frac{8}{3}t^3 \Big|_0^1 + 3t^3 + 3t^2 + 3t \Big|_0^1 = \boxed{\frac{35}{3}}$$

$$\frac{8}{3} + 3 + 3 + 3 =$$

8

(2) Sean C la recta que une a $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ con $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

y $\vec{F} = \begin{pmatrix} xy \\ y^2 \\ e^z \end{pmatrix}$ Halle $\int_C \vec{F} \cdot d\vec{r}$

$$Rot = \begin{vmatrix} i & j & k \\ f_x & f_y & f_z \\ xy & y^2 & e^z \end{vmatrix} = \begin{pmatrix} 0 - 0 \\ -0 - 0 \\ 0 - x \end{pmatrix}$$ No Conservativo

$0 \leq t \leq 1$

$$\left.\begin{aligned} x &= 1 + (t)(1) \\ y &= 1 + (t)(2) \\ z &= 1 + (t)(0) \end{aligned}\right\rangle \quad \begin{aligned} x &= 1+t \\ y &= 1+2t \\ z &= 1 \end{aligned} \qquad r'(t) = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

$$\int_0^1 \begin{pmatrix} (1+t)(1+2t) \\ (1+2t)^2 \\ e^1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} dt \qquad = \int_0^1 10t^2 + 11t + 3$$

$$= \frac{10}{3}t^3 + \frac{11}{2}t^2 + 3t$$

$$\int_0^1 (1+t)(1+2t) + (1+2t)^2 \, dt = \frac{2\,10}{2\,3} + \frac{3\,11}{3\,2} + \frac{3\,6}{6}$$

$$\int_0^1 1 + \cancel{2t} + \cancel{t} + \cancel{2t^2} + 2 + \cancel{8t} + \cancel{8t^2} \, dt = \frac{20}{26} + \frac{33}{6} + \frac{18}{6} \frac{15}{2}$$

$$= \boxed{\frac{71}{6}}$$

③ En las siguientes ~~integrales de linea~~ Ejercicios evalue las I de linea $\int_C \vec{F} \cdot d\vec{r}$ para F y curva C dada por la parametrizacion

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(a) $\vec{F}(x,y) = \begin{pmatrix} xy \\ 3y^2 \end{pmatrix}$ $\vec{r}(t) = \begin{pmatrix} 11t^4 \\ t^2 \end{pmatrix}$ $0 \leq t \leq 1$

$\int_0^1 \begin{pmatrix} xy \\ 3y^2 \end{pmatrix} \cdot \begin{pmatrix} 44t^3 \\ 2t \end{pmatrix} dt$ $r'(t) = \begin{pmatrix} 44t^3 \\ 2t \end{pmatrix}$

$\int_0^1 \begin{pmatrix} 11t^6 \\ 3t^4 \end{pmatrix} \cdot \begin{pmatrix} 44t^3 \\ 2t \end{pmatrix} dt = \frac{484}{10} t^{10} + \frac{6}{7} t^7$

$\int_0^1 484t^9 + 6t^6 \, dt = \boxed{\frac{1724}{35}}$

con $\vec{r}(t) \begin{pmatrix} 11t^4 \\ t^3 \end{pmatrix} \rightarrow$ el resultado es $247/5$

(b) $\vec{F}(x,y,z) = \begin{pmatrix} x+y \\ y-z \\ z^2 \end{pmatrix}$ $\vec{r}(t) = \begin{pmatrix} t^2 \\ t^3 \\ t^2 \end{pmatrix}$ $0 \leq t \leq 1$

$r'(t) = \begin{pmatrix} 2t \\ 3t^2 \\ 2t \end{pmatrix}$

$\int_0^1 \begin{pmatrix} x+y \\ y-z \\ z^2 \end{pmatrix} \cdot \begin{pmatrix} 2t \\ 3t^2 \\ 2t \end{pmatrix} dt$

$\int_0^1 (t^2+t^3)(2t) + (t^3-t^2)(3t^2) + t^4(2t) \, dt$

$\int_0^1 2t^3 + \cancel{2t^4} + 3t^5 - \cancel{3t^4} + 2t^5 \, dt$

$\int_0^1 5t^5 - t^4 + 2t^3 - 6t^2 = \frac{5}{6} + \frac{2}{4} - \frac{1}{5}$

$\frac{5}{6}t^6 - \frac{1}{5}t^5 + \frac{2}{4}t^4 \Big|_0^1 = \boxed{\frac{17}{15}}$

Norma

(c) $\vec{F}(x,y,z) = \begin{pmatrix} \operatorname{Sen} x \\ \cos y \\ xz \end{pmatrix}$ $\quad \vec{r}(t) = \begin{pmatrix} t^3 \\ -t^2 \\ t \end{pmatrix} \quad 0 \leq t \leq 1$

$$\int_0^1 \begin{pmatrix} \operatorname{Sen} x \\ \cos y \\ xz \end{pmatrix} \cdot \begin{pmatrix} 3t^2 \\ -2t \\ 1 \end{pmatrix} dt \qquad r'(t) = \begin{pmatrix} 3t^2 \\ -2t \\ 1 \end{pmatrix}$$

$$\int_0^1 \operatorname{Sen}(t^3) \cdot 3t^2 + 2t\cos(t^2) + (t^3)(t)$$

$$\int_0^1 3t^2 \cdot \operatorname{Sen}(t^3)\,dt + \int_0^1 \cos(t^2) \cdot 2t\,dt + \int_0^1 t^4\,dt$$

$u = (t^3)$ $\quad u = t^2$

$du = 3t^2\,dt$ $\quad du = 2t\,dt$

$$\int_0^1 \operatorname{Sen}(u) + \int_0^1 \cos(u) + \left.\frac{t^5}{5}\right|_0^1$$

$$-\cos(u)\Big|_0^1 + \operatorname{Sen}(u)\Big|_0^1 + \frac{1}{5}$$

$$-\cos(1) + 1 + \operatorname{Sen}(1) + \frac{1}{5}$$

$$\boxed{\frac{6}{5} - \cos(1) + \operatorname{Sen}(1)}$$

(d) $\vec{F}(x,y,z) = \begin{pmatrix} z \\ y \\ x \end{pmatrix}$ $\quad \vec{r}(t) = \begin{pmatrix} t \\ \operatorname{sen} t \\ \cos t \end{pmatrix} \quad 0 \leq t \leq \pi$

$$\int_0^\pi \begin{pmatrix} z \\ y \\ x \end{pmatrix} \cdot \begin{pmatrix} 1 \\ \cos(t) \\ -\operatorname{Sen}(t) \end{pmatrix} \qquad r'(t) = \begin{pmatrix} 1 \\ \cos(t) \\ -\operatorname{Sen}(t) \end{pmatrix}$$

$$\int_0^{\pi} \begin{pmatrix} \cos t \\ \operatorname{sen} t \\ t \end{pmatrix} \cdot \begin{pmatrix} 1 \\ \cos(t) \\ -\operatorname{sen}(t) \end{pmatrix} dt$$

$$\int_0^{\pi} \cos t + \operatorname{sen}(t) \cdot \cos(t) - t \operatorname{sen}(t)\, dt$$

$$\int_0^{\pi} \cos(t) + \frac{\operatorname{sen}(2t)}{2} - \int_0^{\pi} t \operatorname{sen}(t)$$

$\operatorname{sen}(2t) = 2 \operatorname{sen} t \cdot \cos(t)$

$$\cancel{\operatorname{sen}(t) \Big|_0^{\pi}} \to 0 + \cancel{\frac{1}{2} \left(\frac{-\cos(2t)}{2}\right)\Big|_0^{\pi}} \to 0 -$$

LIATE

$u = t$
$dv = \operatorname{sen} t$

$du = 1$
$v = -\cos(t)$

$$\left[-\cos(t) \cdot t + \operatorname{sen}(t)\right|_0^{\pi}$$

$$-\left[(+1(\pi)) - (-1(0) + 0)\right]$$

$-\cos(t) \cdot t - \int -\cos(t)\, du$

$$\boxed{-\pi}$$

$(-\cos(t) \cdot t + \operatorname{sen}(t))$

④ Halle el trabajo realizado por el campo de Fuerzas

$\vec{F}(x, y) = \begin{pmatrix} \frac{y}{1+x^2} \\ 2y \tan^{-1}(x) \end{pmatrix}$

$\vec{r}(t) = \begin{pmatrix} 1t - \operatorname{sen}(t) \\ 1 + \cos(t) \end{pmatrix}$

$$\vec{F} = \begin{pmatrix} x \\ y+2 \end{pmatrix}$$

$0 \le t \le 2\pi$

$M = x\frac{y^2}{}$ $N = y+2$

$My = Nx \to$ Conservativo

$My = 1$ $Nx = 1$

$f_x = \int x \, dx$

$F = \frac{x^2}{2} + g(y)$

$F_y = g'(y)$

$\int y+2 = \int g'(y)$

$= \frac{y^2}{2} + 2y = g(y)$

$\frac{x^2}{2} + \frac{y^2}{2} + 2y$

$F(r(b)) - F(r(a))$

$r_b = \begin{pmatrix} 2\pi \\ 2 \end{pmatrix}$ $r_a = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$\frac{4\pi^2}{2} \left(\cancel{+\frac{4}{2} + 4} \right) - \left(\cancel{\frac{4}{2} + 4} \right)$

$\boxed{2\pi^2}$

5 Halle el trabajo

$\vec{F} \begin{pmatrix} \frac{y^2}{1+x^2} \\ 2y\tan^{-1}(x) \end{pmatrix}$ $\vec{r}(t) = \begin{pmatrix} 2t \\ t^2 \end{pmatrix}$ $0 \le t \le 1$

$M = \dfrac{y^2}{1+x^2}$ $N = 2y\tan^{-1}(x)$

$M_y = N_x =$ Son conservativos

$M_y = \dfrac{2y}{1+x^2}$ $N_x = \dfrac{2y}{1+x^2}$

$\int \dfrac{y^2}{1+x^2}\,dx$

$F = y^2\tan^{-1}(x) + g(y)$

$F_y = 2y\tan^{-1}(x) + g'(x$

$g'(y) = 0$

$g(y) = K$

$y^2\tan^{-1}(x) + 0$

$F(r(b)) - F(r(a))$

$r(b) = \begin{pmatrix}2\\1\end{pmatrix}$ $r(a) = \begin{pmatrix}8\end{pmatrix}$

$1 \cdot \tan^{-1}(2)$

$\boxed{\tan^{-1}(2)}$

⑥ Halle el trabajo realizado por el campo

$\vec{F}(x,y,z) = \begin{matrix} y+z \\ x+z \\ x+y \end{matrix}$ $C = \begin{pmatrix}1\\0\\0\end{pmatrix}$ a $\begin{pmatrix}3\\4\\2\end{pmatrix}$

$Rot = \begin{pmatrix}0\\0\\0\end{pmatrix}$ es conservativo $\begin{vmatrix} i & j & \cancel{k} \\ f_x & F_y & F_z \\ y+z & x+z & x+y \end{vmatrix}$

$$Rot = \begin{pmatrix} 1-1 \\ -(1-1) \\ 1-1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Es conservativo

$F(r(b)) - F(r(a))$

$$\left.\begin{aligned} x &= 1+2t \\ y &= 0+4t \\ z &= 0+2t \end{aligned}\right\} \quad \begin{aligned} x &= 1+2t \\ y &= 4t \quad 0 \leq t \leq 1 \\ z &= 2t \end{aligned}$$

Por F

$f_x = \int y+z \, dx$

$F = xy + zx + g(y,z)$

$f_y = x + g'(y,z)$

$\int z = \int g'(y,z)$

$g(y) = zy + g(z)$

$\underline{F = xy + zx + zy + g(z)}$

$f_z = x + y + g'(z)$

$x+y = x+y + g'(z)$

$g(z) = K \to 0$

$F(r(b)) - F(r(a))$

$r(b) = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \quad r(a) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

Directo

$$\int_0^1 \begin{pmatrix} 6t \\ 1+4t \\ 1+6t \end{pmatrix} \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix} dt$$

$$\int_0^1 12t + 4 + 16t + 2 + 12t \, dt$$

$$\int_0^1 40t + 6 \, dt$$

$$\frac{40t^2}{2} + 6t$$

$\boxed{26}$

$12 + 6 + 8 - 0$

$\boxed{26}$

7) Encuentre la integral de linea del campo vectorial

$$\vec{F}(x,y,z) = \begin{pmatrix} \cos(x+y) \\ 2yz\, e^{y^2} + \cos(x+y) \\ \cancel{\cos(x+y)} \quad y^2\, e^{y^2 z} \end{pmatrix}$$

$$\vec{r}(t) = \begin{pmatrix} \operatorname{Sen}(40t) \\ (2+\cos(40t)) \cdot \cos(t) \\ (2+\cos(40t)) \cdot \operatorname{sen}(t) \end{pmatrix} \qquad 0 \le t \le \pi$$

8 Considere el campo vectorial

$$\vec{F}(x,y,z) = \begin{pmatrix} \dfrac{y e^{xy}}{1+e^{2xy}} \\ \dfrac{x e^{xy}}{1+e^{2xy}} + z^3 \\ 3yz^2 \end{pmatrix}$$

$$\vec{r}(t) = \begin{pmatrix} (2+\cos(8t))\cos t \\ (2+\cos(8t))\,\text{sen}\,t \\ \text{Sen}\,(8t \end{pmatrix} \quad 0 \leq t \leq 2\pi$$

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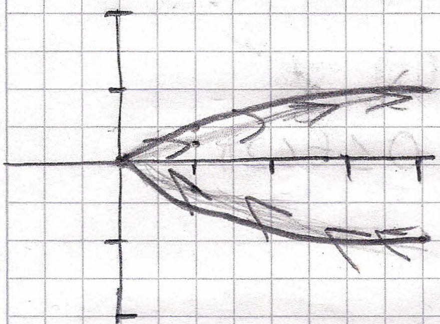
9) Sea $C = \{(x,y) \in \mathbb{R}^2 : 4y^2 = x,\ x \leq 4$ orientado en sentido horario

Calcule

$$\int_C \overset{M}{(\tan^{-1}(y))}\,dx + \overset{N}{\left(\frac{x}{1+y^2} + 2ye^{y^2}\right)}\,dy$$



$y = t$

$x = 4t^2$

$-1 \leq t \leq 1$

Si $M_y = N_x$ es conservativo

$M_y = \frac{1}{1+y^2}$ $\quad N_x = \frac{1}{1+y^2}$

Hallemos F

$f_x = \int \tan^{-1}(y)\,dx$

$F = x \cdot \tan^{-1}(y) + g(y)$

$F_y = \frac{x}{1+y^2} + g'(y) = $

$= \int 2y\,e^{y^2} = \int g'(y)$

$u = y^2$

$du = 2y$

$g(y) = e^{y^2}$ ✓

$F = x \cdot \tan^{-1}(y) + e^{y^2}$

$r(b) = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ $\quad r(a) = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

$4 \tan^{-1}(1) + e^1 - \left[4 \cdot \tan^{-1}(-1) + e^1 \right]$

$4 \tan^{-1}(1) + \cancel{e^1} + 4 \tan^{-1}(1) \cancel{- e}$

$8 \tan^{-1}(1)$

$8 \cdot \frac{1}{4} \pi$ $\quad \boxed{2\pi}$

(10) Cual de los siguientes campos vectoriales es conservativo , si es conservativo halle F.

(a) $\vec{F}(x,y) = \overset{M}{(2x - 3y)} \hat{i} + \overset{N}{(-3x + 4y - 8)} \hat{j}$

$\begin{pmatrix} 2x - 3y \\ -3x + 4y - 8 \end{pmatrix} = \begin{pmatrix} f_x \\ f_y \end{pmatrix}$ si $M_y = N_x$ Entonces es conservativo

$M_y = -3$
$N_x = -3$

$\int 2x - 3y \, dx$

$F = x^2 - 3xy + g(y)$

$-3x + g'(y) = F_y$

$\cancel{-3x} + 4y - 8 = \cancel{-3x} + g'(y)$

$\int g'(y) = \int 4y - 8 \, dy$

$g(y) = 2y^2 - 8y$

$$\boxed{F = x^2 - 3xy + 2y^2 - 8y}$$

(b) $\vec{F}(x,y) = \begin{pmatrix} e^x \text{ sen } y \\ e^x \cos y \end{pmatrix} = \begin{matrix} M \\ N \end{matrix} = \begin{matrix} F_x \\ F_y \end{matrix}$

$M_y = N_x$ Conservativo

$M_y = e^x \cos y$

$N_x = e^x \cos y$

$\int e^x \text{ sen}(y)\, dx$

$F = e^x \text{ sen}(y) + g(y)$

$e^x \cos(y) + g'(y) = f_y$

$g'(y) = 0$

$g(y) = K \rightarrow 0$

$$\boxed{F = e^x \text{ sen}(y)}$$

(c) $\vec{F}(x,y) = \begin{pmatrix} e^x \cos y \\ e^x \text{ sen } y \end{pmatrix} = \begin{matrix} M \\ N \end{matrix}$

$M_y = N_x \rightarrow$ conservativo

$M_y = -e^x \text{ sen}(y)$

$N_x = e^x \text{ sen}(y$

No es conservativo

(D) $\vec{F}(x,y) = \begin{pmatrix} 3x^2 - 2y^2 \\ 4xy + 3 \end{pmatrix}$

$M_y = N_x \rightarrow$ Conservativo

$M_y = -4y$
$N_x = 4y$ No es conservativo

(E) $\vec{F}(x,y) = \begin{pmatrix} e^x \operatorname{sen}(y) - \frac{1}{x^2+1} + y \\ e^x \cos(y) + 2x - 3y^2 \end{pmatrix} = \begin{matrix} M \\ N \end{matrix}$

$M_y = N_x \to$ conservativo

$M_y = e^x \cos(y) + 1$
$N_x = e^x \cos(y) + 2$ No es conservativo

(F) $\vec{F}(x,y) = \begin{pmatrix} e^x \operatorname{sen}(y) - \frac{1}{x^2+1} + y \\ e^x \cos(y) - 3y^2 \end{pmatrix} = \begin{matrix} M \\ N \end{matrix}$

$M_y = N_x \to$ conservativo

$M_y = e^x \cos(y) + 1$
$N_x = e^x \cos(y)$ No es conservativo

(6) $\vec{F}(x,y) = \begin{pmatrix} e^x \operatorname{sen}(y) + y \\ e^x \cos(y) + 2x \end{pmatrix} = \begin{matrix} M \\ N \end{matrix}$

$M_y = N_x \to$ Conservativo

$M_y = e^x \cos(y) + 1$
$N_x = e^x \cos(y) + 2$ No es conservativo

(H) $\vec{F}(x,y) = \begin{pmatrix} e^x \operatorname{sen} y - \frac{1}{x^2+1} \\ e^x \cos y - 3y^2 \end{pmatrix} = \begin{matrix} M \\ N \end{matrix} = \begin{matrix} f_x \\ f_y \end{matrix}$

$M_y = N_x \Rightarrow$ Conservativo

$M_y = e^x \cos(y)$
$N_x = e^x \cos(y)$ Conservativo

$\int e^x \cos(y) - 3y^2 \, dy$

$F = e^x \operatorname{sen}(y) - y^3 + g(x)$

$F_x = e^x \operatorname{sen}(y) + g'(x)$

$\int -\frac{1}{x^2+1} = \int g'(x)$

$g(x) = -\tan^{-1}(x)$

$\boxed{F = e^x \operatorname{sen}(y) - y^3 - \tan^{-1}(x)}$

(11) En los siguientes ejercicios, encuentre la funcion F tal que $\vec{F} = \nabla F$ y use el resultado para calcular a lo largo de la curva C

$\int_C \vec{F} \cdot \vec{dr}$

(a) $\vec{F}(x,y) = \begin{pmatrix} x^2 \\ y^2 \end{pmatrix} = \begin{matrix} M \\ N \end{matrix}$ C: Es el arco de la parabola $y = 2x^2$ de $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ a $\begin{pmatrix} 2 \\ 8 \end{pmatrix}$

$M_y = N_x \Rightarrow$ conservativo

$M_y = 0$

$N_x = 0$

Hallemos F

$\int x^2\, dx$

$F = \frac{x^3}{3} + g(y)$

$F_y = 0 + g'(y)$

$\int y^2 = \int g'(y)$

$g(y) = \frac{y^3}{3}$

$$\boxed{F = \frac{x^3}{3} + \frac{y^3}{3}}$$

$F(r(b)) - F(r(a))$

$\frac{\cancel{8}}{\cancel{3}} + \frac{512}{3} - \left[\frac{-1}{3} + \frac{\cancel{0}}{\cancel{3}} \right]$

$\frac{512}{3} + \frac{1}{3}$

$$\boxed{171}$$

(b) $\vec{F}(x,y) = \begin{pmatrix} xy^2 \\ x^2y \end{pmatrix} = \begin{pmatrix} M \\ N \end{pmatrix}$ C: $\vec{r}(t) = \begin{pmatrix} t + \text{sen}(\pi t/2) \\ t + \cos(\pi t/2) \end{pmatrix}$

$M_y = N_x \Rightarrow$ conservativo

$0 \le t \le 1$

$M_y = 2xy$
$N_x = 2xy$

$F = \frac{x^2y^2}{2} + g(y)$

$r(b) = \begin{matrix} 1+0 \\ 1+1 \end{matrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\int xy^2\, dx$

$F_y = yx^2 + g'(y)$

$g(y) = K \to 0$

$r(a) = \begin{matrix} 0+0 \\ 0+1 \end{matrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$F = \frac{x^2 y^2}{2}$

$F(r(b)) - F(r(a))$

$\frac{1 \cdot 4}{2} - 0$

$\boxed{2}$

c) $\vec{F}(x,y) = \begin{pmatrix} \frac{y^2}{1+x^2} \\ 2y \arctan(x) \end{pmatrix}$

$C: \vec{r}(t) = \begin{pmatrix} t^2 \\ 2t \end{pmatrix}$

$0 \leq t \leq 1$

$M_y = N_x$

$M_y = \frac{2y}{1+x^2}$

$N_x = \frac{2y}{1+x^2}$

$r(b) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $r(a) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$F = y^2 \cdot \tan^{-1}(x)$

Hallemos F

$F(r(b)) - F(r(a))$

$\int \frac{y^2}{1+x^2} dx$

$4 \cdot \frac{\pi}{4} - 0$

$F = y^2 \tan^{-1}(x) + g(y)$

$\boxed{\pi}$

$F_y = 2y \tan^{-1}(x) + g'(y)$

$2y \tan^{-1}(x) = 2y \tan^{-1}(x) + g'(y)$

$g(y) = K \rightarrow 0$

(d) $\vec{F}(x,y,z) = \begin{pmatrix} yz \\ xz \\ xy+2z \end{pmatrix} = \begin{matrix} f_x \\ f_y \\ f_z \end{matrix}$ C= Segmento que va de $\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ a $\begin{pmatrix} 4 \\ 6 \\ 3 \end{pmatrix}$

Rot = 0 $\rightarrow$ conservativo

$$\begin{vmatrix} i & j & k \\ f_x & f_y & f_z \\ yz & xz & xy+2z \end{vmatrix} = \begin{pmatrix} x-x \\ -(y-y) \\ z-z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \checkmark$$

$\int yz \, dx$

$F = xyz + g(y,z)$

$F_y = xz + g'(y,z)$

$xz = xz + g'(y,z)$

$\int g'(y,z) = \int 0 \rightarrow 0$

$g(y,z) = k + g(z)$

$F = xyz + 0 + g(z)$

$F_z = xy + g'(z)$

$xy + 2z = xy + g'(z)$

$\int 2z = \int g'(z)$

$g(z) = z^2$

$F = xyz + z^2$

$F(r(b)) - F(r(a))$

$4 \cdot 6 \cdot 3 + 9 - [4]$

$4 \cdot 6 \cdot 3 + 5$

$72 + 5$

$\boxed{77}$

(12) Muestre que el valor de la integral de linea

$$\int_C \overset{M}{(\tan y)}\,dx + \overset{N}{(x \sec^2(y))}\,dy$$

es el mismo para cualquier trayectoria que una $\begin{pmatrix}1\\0\end{pmatrix}$ con $\begin{pmatrix}2\\\pi/4\end{pmatrix}$

$M_y = N_x \rightarrow$ Conservativo

$M_y = \sec^2(y)$

$N_x = \sec^2(y)$

$\hookrightarrow$ Si es conservativo implica que C solo es una trayectoria es decir que solo hay una curva C ~~(la de la curva)~~ que esta entre los dos puntos

Para comprobarlo hagamoslo de las dos maneras

$F = x \tan y$

Evaluemos en los extremos

$F(r(b)) - F(r(a))$

$2\tan(\pi/4) - 1\tan(0)$

$\boxed{2}$

Y por el otro lado

$0 \leq t \leq 1$

$x = 1 + (t)(1)$ $\qquad x = t + 1$

$y = 0 + (t)(\pi/4)$ $\qquad y = \dfrac{t\pi}{4}$

$$v'(t) = \begin{pmatrix} 1 \\ \pi/4 \end{pmatrix}$$

$$\int_0^1 \begin{pmatrix} \tan y \\ x \sec^2(y) \end{pmatrix} \cdot \begin{pmatrix} 1 \\ \pi/4 \end{pmatrix} dt$$

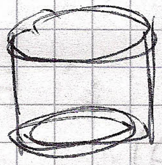
$$\int_0^1 \tan\left(\frac{t\pi}{4}\right) + \frac{\pi}{4}(t+1)\sec^2\left(\frac{t\pi}{4}\right)$$

13

$$\bar{r}(u,v) = \begin{pmatrix} v \operatorname{sen} u \\ v \cos u \\ u \end{pmatrix} \quad \text{III}$$

Rollo

$$\vec{r}(u,v) = \begin{pmatrix} 2 \operatorname{sen} u \\ 3 \cos u \\ v \end{pmatrix} \quad \text{VI}$$



Cilindro

$$\vec{r}(s,t) = \begin{pmatrix} s \\ t \\ t^2 - s^2 \end{pmatrix} \quad \text{II}$$

Silla de montar sobre ~~el~~

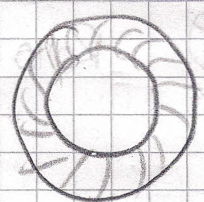
$$z = y^2 - x^2$$

$$\vec{r}\,(s,t) = \begin{pmatrix} s\ \text{sen}(2t) \\ s^2 \\ s\cos(2t) \end{pmatrix}$$

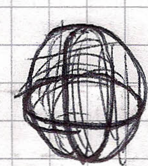
$$y^2 = x^2 + z^2$$

Una parte del hiperboloide de dos hojas

$$\vec{r}\,(\phi,\theta) = \begin{pmatrix} 4\cos\theta + \cos\phi \cdot \cos\theta \\ 4\,\text{sen}\,\theta + \cos\phi \cdot \cos\theta \\ \text{sen}\,\phi \end{pmatrix}$$



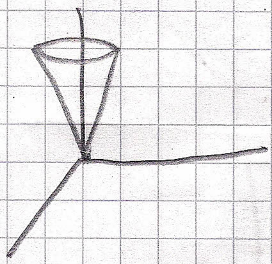
$$\vec{r}\,(\phi,\theta) = \begin{pmatrix} \text{sen}\,\phi \cdot \cos\theta \\ \text{sen}\,\phi\ \text{sen}\,\theta \\ \cos\phi \end{pmatrix} \quad V$$

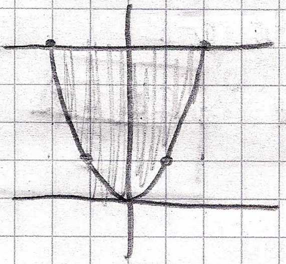


Esfera

(14) Halle el area Superficial de la porcion del cono

$z\sqrt{x^2+y^2}$ que esta entre el plano $y=4$ y el cilindro $y=x^2$





$x^2 \leq y \leq 4$

$-2 \leq x \leq 2$

La Parametrizacion mas simple es

$x=u$
$y=v$
$z=\sqrt{u^2+v^2}$

↳ Cuando Parametrizamos asi

$\|r_u \times r_v\| = \sqrt{(f_x)^2+(f_y)^2+1}$

Hallemos

$f_x = \dfrac{x}{\sqrt{x^2+y^2}}$

$f_y = \dfrac{y}{\sqrt{x^2+y^2}}$

La region de integracion

$\int_{-2}^{2}\int_{x^2}^{4} \|r_u \times r_v\|$

$\int_{-2}^{2}\int_{x^2}^{4} \sqrt{\left(\dfrac{x}{\sqrt{x^2+y^2}}\right)^2+\left(\dfrac{y}{x^2+y^2}\right)+1}$

$\int_{-2}^{2}\int_{x^2}^{4} \sqrt{2}\, dy\, dx$

$\sqrt{2}\int_{-2}^{2} (4-x^2)\, dx$

$2\sqrt{2}\int_{-2}^{2} 4-x^2$

$2\sqrt{2}\;\; 4x - \dfrac{x^3}{3}\Big|_{-2}^{2}$

$$2\sqrt{2}\left[8-\frac{8}{3}\right]-\left[-8+\frac{8}{3}\right]$$

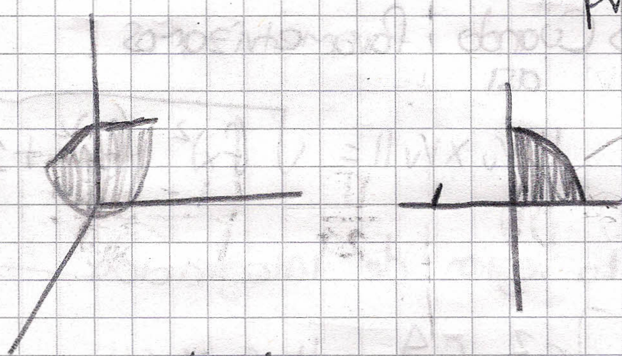
$$8-\frac{8}{3}+8-\frac{8}{3}$$

$$16-\frac{16}{3} \qquad \boxed{\frac{64\sqrt{2}}{3}}$$

(15) Halle el are superficial de la porcion del paraboloide

$z = x^2 + y^2$ debajo de $z=4$

Primer octante



$x^2+y^2=4$

$x=\sqrt{4-y^2}$

$x = u$
$y = v$
$z = u^2+v^2$ $\rightarrow$ $\|r_u \times r_v\|$

$$\sqrt{(2x)^2+(2y)^2+1}$$

$$\sqrt{4x^2+4y^2+1}$$

$$4(x^2+y^2)+1$$

$$\int_0^2 \int_0^{\sqrt{4-y^2}} \sqrt{4(x^2+y^2)+1}\; dx\, dy$$

Polares

$x = r\cos\theta$
$y = r\,\text{sen}\,\theta$

$$\int_0^{\pi/2} \int_0^2 \sqrt{4r^2+1}\; r\, dr\, d\theta$$

$u = 4r^2+1$
$du = 8r$

$$\frac{1}{8} \int_0^{\pi/2} \int_1^{17} \sqrt{u}\; du\, d\theta$$

$$\frac{2\pi}{8 \cdot 3} \cdot \frac{1}{2} \left[\sqrt{u^3} \Big|_1^{17} \right]$$

$$\boxed{\frac{\pi}{24} \left[\sqrt{17^3} - 1 \right]}$$

$$\frac{\pi}{24} \left(\frac{1 - \sqrt{17}}{2\sqrt{17}} \right)$$

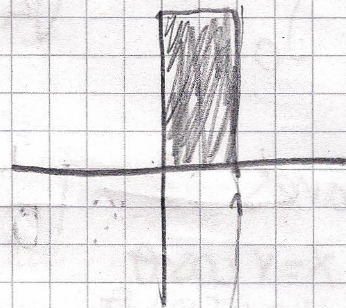
$u^{1/2+}$

16) Halle el area superficial con Ecuaciones parametricas

$x = u^2$

$y = uv$ $\qquad 0 \leq u\ 1$

$z = \frac{1}{2}v^2$ $\qquad 0 \leq v \leq 2$



$$\iint_D \| r_u \cdot r_v \|$$

$$r_u = \begin{pmatrix} 2u \\ v \\ 0 \end{pmatrix}$$

$$\begin{vmatrix} i & j & k \\ 2u & v & 0 \\ 0 & u & v \end{vmatrix} = \begin{pmatrix} v^2 \\ -2uv \\ 2u^2 \end{pmatrix} \qquad r_v = \begin{pmatrix} 0 \\ u \\ v \end{pmatrix}$$

$$\int_0^2 \int_0^1 \sqrt{v^4 + 4u^2v^2 + 4u^4}\ du\, dv$$

$$\int_0^2 \int_0^1 \sqrt{(v^2 + 2u^2)^2}\ du\, dv$$

$$\int_0^2 \int_0^1 v^2 + 2u^2\ du\, dv$$

$$\int_0^2 uv^2 + \frac{2}{3}u^3 \Big|_{u=0}^{u=1}$$

$$\int_0^2 v^2 + \frac{2}{3} \, dv$$

$$\frac{v^3}{3} + v\frac{2}{3} \Big|_0^2$$

$$\frac{8}{3} + \frac{4}{3}$$

$$\frac{12}{3} = \boxed{4}$$

(17) Calcule el are superficial de la esfera $(x-a)^2 + (y-b)^2 + (z-c)^2 = R^2$

(18) Parametrice la Superficie S dada por

$$S = (x, y, z) \mid z = 4 - x^2 - y^2$$

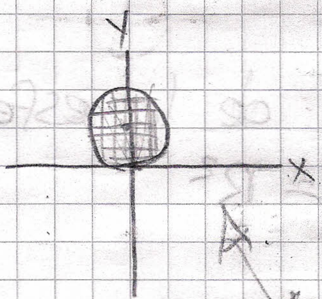
$$x^2 + (y-1)^2 \leq 1$$

y Plantee el area

$X = u$
$Y = v$

$Z = 4 - u^2 - v^2$

$$\|r_u \times r_v\| = \sqrt{(-2x)^2 + (-2y)^2 + 1}$$



A Polares

$$\sqrt{4x^2 + 4y^2 + 1}$$

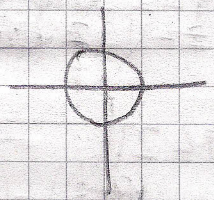
$X = r\cos\theta$
$Y = r\,\text{sen}\,\theta$

$$\int_0^{\pi} \int_0^{2\,\text{sen}\,\theta} \sqrt{4r^2+1}\; r\,dr\,d\theta$$

(19) Halle el area superficial de la esfera $x^2+y^2+z^2=a^2$ que se encuentra dentro del cilindro $x^2+y^2=ax$

$x = a \operatorname{sen}\phi \cos\theta$
$y = a \operatorname{sen}\phi \operatorname{sen}\theta$
$z = a\cos\phi$

$$\iint_{\Sigma} F \, ds = \iint_{\Sigma} \vec{F} \cdot \vec{n} \, ds$$



$0 \le \phi \le \pi$
$0 \le \theta \le 2\pi$

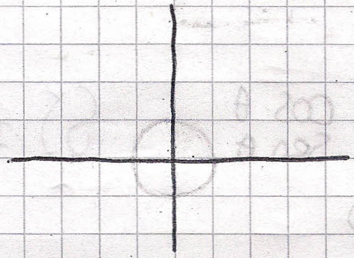
$$r_\phi = \begin{pmatrix} a\cos\phi\cos\theta \\ a\cos\phi \operatorname{sen}\theta \\ -a\cos\phi \end{pmatrix}$$

$$r_\theta = \begin{pmatrix} -a \operatorname{sen}\phi \operatorname{sen}\theta \\ a \operatorname{sen}\phi \cos\theta \\ 0 \end{pmatrix}$$

$$\int_0^{\pi}\int_0^{2\pi} a^2 \operatorname{sen}\phi \, d\theta \, d\phi$$

(20) $\iint_S y \, ds$ donde S es la parte de la esfera $x^2+y^2+z^2=4$ que esta dentro del cilindro $x^2+y^2=1$ y arriba del plano xy

$\cancel{z=\sqrt{4-x^2-y^2}}$



$x = 2 \operatorname{sen}\phi \cos\theta$
$y = 2 \operatorname{sen}\phi \operatorname{sen}\theta$
$z = \sqrt{\ } \; 2\cos\phi$

$1 + z^2 = 4$

$z^2 = 4 - 1$

$z = \sqrt{3}$

$2\cos\phi = \sqrt{3}$

$\cos\phi = \frac{\sqrt{3}}{2}$

$\phi = \pi/6$

$0 \leq \theta \leq 2\pi$

$0 \leq \phi \leq \frac{\pi}{6}$

$$\int_0^{2\pi}\int_0^{\pi/6} 2\operatorname{sen}\phi \operatorname{sen}\theta \cdot 4\operatorname{sen}\phi$$

$$\frac{8}{2}\int_0^{2\pi}\int_0^{\pi/6} \operatorname{sen}^2\phi \cdot \operatorname{sen}\theta \, d\phi \, d\theta$$

$$4\int_0^{2\pi} \operatorname{sen}\theta \int_0^{\pi/6} 1 - \cos(2\phi) \, d\phi \, d\theta$$

$-4\left[\cancel{1-1}\right] \cdot \left[\phi - \frac{\operatorname{sen}(2\phi)}{2}\Big|_0^{\pi/6}\right]$ DO

$\boxed{0}$

21 Evalue $\iint_S xyz\, dS$ donde S es el cono con ecuaciones parametricas

$x = u+v$

$y = u-v$

$z = 1+2u+v$

$0 \le u \le 1$

$0 \le v \le \pi$

$r_u = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ $\quad v_u = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

$\left|\begin{matrix} i & j & k \\ 1 & 1 & 2 \\ 1 & -1 & 1 \end{matrix}\right| = \begin{pmatrix} 1+2 \\ -(1-2) \\ -1-1 \end{pmatrix} = \left\|\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}\right\|$

$9+1+4 \qquad \|v_u \times r_u\| = \sqrt{14}$

$\sqrt{14}\int_0^{\pi}\int_0^1 (u^2-v^2)(1+2u+v)\, du\, dv$

$\sqrt{14}\int_0^{\pi}\int_0^1 u^2+2u^3+u^2v-v^2-2v^2u-v^3$

$\frac{u^3}{3}+\frac{u^4}{2}+\frac{u^3v}{3}-uv^2-v^2u^2-uv^3$

$\frac{1}{3}+\frac{1}{2}+\frac{v}{3}-v^2-v^2-v^3$

$$\sqrt{14}\int_0^{\pi} \frac{5}{6}+\frac{v}{3}-2v^2-v^3 \; dv$$

$$\frac{5v}{6}+\frac{v^2}{6}-\frac{2v^3}{3}-\frac{v^4}{4}\Big|_0^{\pi}$$

$$\sqrt{14}\left[\frac{5\pi}{6}+\frac{\pi^2}{6}-\frac{2\pi^3}{3}-\frac{\pi^4}{4}\right]$$

$$\sqrt{14}\left[\frac{5\pi+\pi^2}{6}-\frac{2\pi^3}{3}-\frac{\pi^4}{4}\right]$$

$$\sqrt{14}\left[\frac{10\pi+2\pi^2}{12}-\frac{8\pi^3}{12}-\frac{3\pi^4}{12}\right]$$

$$\frac{\sqrt{14}}{12}\left[10\pi+2\pi^2-8\pi^3-3\pi^4\right]$$

22 ~~Evalue $\iint_S (x^2+y^2)\,ds$ donde S es la región triangular con vértices $(1,0,0)$ $(0,2,0)$ $(0,0,2)$~~

Evalue $\iint_S (x^2+y^2)\,ds$ donde S es la superficie con ecuación vectorial

$$r(u,v)=\begin{pmatrix}2uv\\ u^2-v^2\\ u^2+v^2\end{pmatrix} \qquad u^2+v^2\leq 1$$

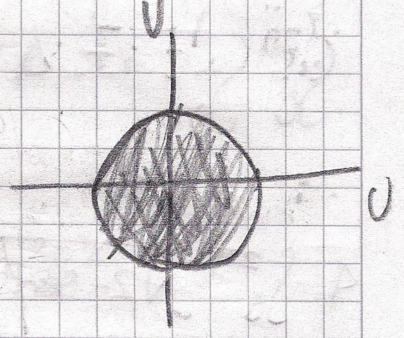
$r_u = \begin{pmatrix} 2v \\ 2u \\ 2u \end{pmatrix} \qquad r_v = \begin{pmatrix} 2u \\ -2v \\ 2v \end{pmatrix} \qquad \begin{vmatrix} i & j & k \\ 2v & 2u & 2u \\ 2u & -2v & 2v \end{vmatrix}$

$4uv + 4uv$
$-(4v^2 - 4u^2)$
$-4v^2 - 4u^2$

$$\sqrt{64u^2v^2 + (4u^2-4v^2)^2 + (4v^2-4u^2)^2} \qquad \left\| \begin{pmatrix} 8uv \\ 4u^2-4v^2 \\ -4v^2-4u^2 \end{pmatrix} \right\|$$

$$\sqrt{\cancel{64}u^2v^2 + \cancel{16u^4} - \cancel{32u^2v^2} + 16v^4 + 16v^4 - \cancel{32u^2v^2} + \cancel{16u^4}}$$

$$\sqrt{32u^4 + 32v^4}$$



$$4u^2v^2 + (u^2-v^2)^2$$

$$4u^2v^2 + u^4 - 2u^2v^2 + v^4$$

$$\iint_D (2u^2v^2 + u^4 + v^4)\sqrt{32u^4 + 32v^4}$$

Polares

$u = r\cos\theta$
$v = r \operatorname{sen}\theta$

$$\int_0^{2\pi}\int_0^1 \left(2r^4\cos^2\theta \operatorname{sen}^2\theta + r^4\cos^4\theta + r^4\operatorname{sen}^4\theta\right)$$
$$\sqrt{32\left(r^4\cos^4\theta + r^4\operatorname{sen}^4\theta\right)}$$
$$r\,dr\,d\theta$$

$$r^6 \sqrt{32(\cos^4\theta + sen^4\theta)}\; r\,dr\,d\theta$$

$$r^7\, 4\sqrt{2(\cos^4\theta + sen^4\theta)}\; dr\,d\theta$$

$$r^7\, 4\sqrt{2(1 - 2\cos^2 sen^2\theta)}$$

$$\int_0^{2\pi}\int_0^1 r^7\, 4\sqrt{2 - sen^2(2\theta)}\; dr\,d\theta$$

$$4\int_0^{2\pi} \sqrt{2 - sen^2(2\theta)} \int_0^1 r^7\, dr\,d\theta$$

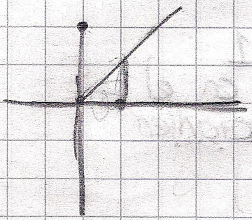
$$\frac{4}{8}\int_0^{2\pi} \sqrt{2 - sen^2(2\theta)}\; d\theta$$

$$\frac{4}{8}\, 2\pi\sqrt{2}$$

Si $\cos^4\theta + sen^4\theta = 1$

$$\boxed{\pi\sqrt{2}}$$

(23) Evalue $\iint_S xy \, dS$ donde S es la region triangular con vertices $\begin{pmatrix}1\\0\\0\end{pmatrix}$ $\begin{pmatrix}0\\2\\0\end{pmatrix}$ $\begin{pmatrix}0\\0\\2\end{pmatrix}$



$2x + y + z = 2$

$$2x + y + z = 2$$

$$z = 2 - 2x - y$$

$$dS = \sqrt{(-2)^2 + (-1)^2 + 1}$$

$$dS = \sqrt{6}$$

$$\sqrt{6}\int_0^1 \int_0^{2-2x} xy \; dy\, dx$$

$2x + y = 2$

$y = 2 - 2x$

$$x\frac{y^2}{2}\Big|_0^{2-2x} \qquad \frac{x^2}{2} - \frac{x^3}{3}\Big|_0^1$$

$$\frac{x(2-2x)}{2}$$

$$\sqrt{6}\int_0^1 x - x^2 \, dx \qquad = \sqrt{6}\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$\frac{\sqrt{6}}{\sqrt{6}} \quad \frac{1}{6} \quad \sqrt{6} \qquad \frac{6}{6\sqrt{6}}$$

$$\boxed{\frac{1}{\sqrt{6}}}$$

(24) Evalue $\iint_S xy\, ds$ donde

S es la parte del plano

$z = 1 - x - y$
$x = x$
$y = y$

$x + y + z = 1$
que esta en el primer octante

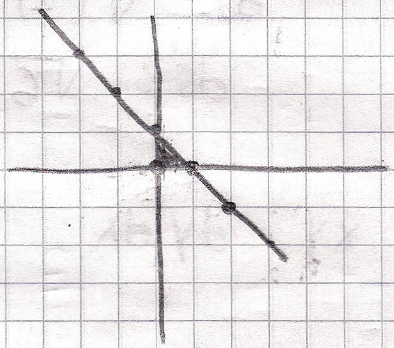
$$ds = \sqrt{(-1)^2 + (-1)^2 + (1)}$$

$x + y = 1$

$$ds = \sqrt{3}$$

$y = 1 - x$

$$\sqrt{3}\int_0^1 \int_0^{1-x} xy \; dy\, dx$$



$$\sqrt{3}\int_0^1 \left.\frac{xy^2}{2}\right|_{y=0}^{1-x} dx$$

$$\sqrt{3}\left[\frac{x(1-x)}{2}\right] = \sqrt{3}\left[\frac{1}{2}\int_0^1 x - x^2 \; dx\right]$$

$$\sqrt{3}\left[\frac{x - x^2}{2}\right] \qquad \sqrt{3}\,\frac{1}{2}\left(\left.\frac{x^2}{2} - \frac{x^3}{3}\right|_0^1\right)$$

$$\sqrt{3}\;\frac{1}{2}\left[\frac{1}{2} - \frac{1}{3}\right]$$

$\sqrt{3}\quad \frac{1}{2}\quad \frac{1}{6}$ $\qquad \boxed{\frac{\sqrt{3}}{12}}$

(25) Evalue $\iint_S y\, ds$ donde s

$z = \frac{2}{3}\left(x^{3/2} + y^{3/2}\right)$

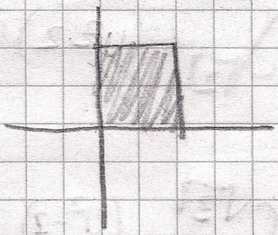
$x = x \qquad y = y$

$0 \le x \le 1$

$0 \le y \le 1$

$ds = \sqrt{(\sqrt{x})^2 + (\sqrt{y})^2 + 1}$

$ds = \sqrt{x+y+1}$



$\int_0^1 \int_0^1 y\sqrt{x+y+1}\; dx\, dy$

$u = x+y+1$

$du = 1$

$\int_0^1 y \int_{y+1}^{y+2} \sqrt{u}\; dx\, dy$

$\frac{2}{3}\int_0^1 y \quad u^{3/2}\Big|_{u=y+1}^{u=y+2}$

$\frac{2}{3}\int_0^1 y\,(y+2)^{3/2}\, dy - \frac{2}{3}\int_0^1 y\,(y+1)^{3/2}\, dy$

Sustitucion

$u = \sqrt{y+2}$

$u^2 = y+2$

$y = u^2 - 2$

$dy = 2u\,du$

Sustitucion

$w = \sqrt{y+1}$

$w^2 = y+1$

$w^2 - 1 = y$

$dy = 2w\,dw$

$$\frac{2}{3}\int_{\sqrt{2}}^{\sqrt{3}} (u^2-2)\, u^3\, 2u\,du - \frac{2}{3}\int_{1}^{\sqrt{2}} (w^2-1)\, w^3\, 2w\,dw$$

$$\frac{2}{3}\int_{\sqrt{2}}^{\sqrt{3}} (u^2-2)\cdot 2u^4 - \frac{2}{3}\int_{1}^{\sqrt{2}} (w^2-1)\, 2w^4$$

$$\frac{2}{3}\int_{\sqrt{2}}^{\sqrt{3}} 2u^6 - 4u^4\,du - \frac{2}{3}\int_{1}^{\sqrt{2}} 2w^6 - 2w^4\,dw$$

$$\frac{4}{3}\ \frac{u^7}{7} - \frac{2u^5}{5}\Big|_{\sqrt{2}}^{\sqrt{3}} - \frac{4}{3}\ \frac{w^7}{7} - \frac{w^5}{5}\Big|_{1}^{\sqrt{2}}$$

$$\frac{4}{3}\left[\frac{(\sqrt{3})^7}{7} - \frac{2(\sqrt{3})^5}{5} - \frac{(\sqrt{2})^7}{7} + \frac{2(\sqrt{2})^5}{5} - \frac{(\sqrt{2})^7}{7} + \frac{(\sqrt{2})^5}{5} + \frac{1}{7} - \frac{1}{5}\right]$$

$$\boxed{\frac{4}{105}\left(-2 + 4\sqrt{2} + 9\sqrt{3}\right)}$$

(26) Halle el flujo del campo vectorial

$$\vec{F}(x,y,z) = \begin{pmatrix} -y \\ x \\ e^{x^2 z^2} \end{pmatrix}$$ a traves de la superficie

$S = 4x^2 + y^2 = 4$

$0 \leq z \leq 3$

$$\iint \vec{F} \, \vec{ds}$$

$\vec{ds} = r_\theta \times r_v$

$r_z = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $r_\theta = \begin{pmatrix} -2\,sen\,\theta \\ 2\cos\theta \\ 0 \end{pmatrix}$

$x = 2\cos\theta$
$y = 2\,sen\,\theta$
$z = z$

$$\begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ -2\,sen\,\theta & 2\cos\theta & 0 \end{vmatrix} \begin{pmatrix} -2\cos\theta \\ +2\,sen\,\theta \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -2\,sen\,\theta \\ 2\cos\theta \\ e^{16\cos\theta \cdot sen\theta} \end{pmatrix} \begin{pmatrix} -2\cos\theta \\ 2\,sen\,\theta \\ 0 \end{pmatrix}$$

$4\,sen\theta\cos\theta + 2\cos\theta\,sen\,\theta$

$$\int_0^{2\pi}\int_0^3 3\,sen(2\theta)\,dz\,d\theta$$

$$\int_0^{2\pi} 9\,sen(2\theta)\,d\theta$$

$9\int_0^{2\pi} Sen(2\theta)\, d\theta$ $\qquad (-1+1)$

$\frac{9}{2} \quad -\cos(2\theta)\Big|_0^{2\pi} \qquad \frac{9}{2} \quad 0$

$$\boxed{0}$$

(27) Una helicoide se define como $\varphi : D \to R^3$

donde $\varphi(r,\theta)$ $\begin{pmatrix} r\cos\theta \\ r\,sen\,\theta \\ \theta \end{pmatrix}$

y D $\quad 0 \leq \theta \leq 2\pi$

$0 \leq r \leq 1$

(a) Halle su area

$\iint_D \vec{F} \cdot d\vec{S} \qquad \| r_r \times r_\theta \|$

$r_r = \begin{pmatrix} \cos\theta \\ sen\,\theta \\ 0 \end{pmatrix} \qquad r_\theta = \begin{pmatrix} -r\,sen\,\theta \\ r\cos\theta \\ 1 \end{pmatrix}$

$$\begin{vmatrix} i & j & k \\ \cos\theta & \operatorname{sen}\theta & 0 \\ -r\operatorname{sen}\theta & r\cos\theta & 1 \end{vmatrix} = \begin{pmatrix} \operatorname{sen}\theta \\ -(\cos\theta) \\ r\cos^2\theta + r\operatorname{sen}^2\theta \end{pmatrix}$$

$$\left\| \begin{pmatrix} \operatorname{sen}\theta \\ -\cos\theta \\ r \end{pmatrix} \right\|$$

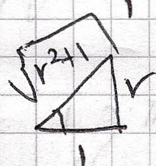
$$\sqrt{\operatorname{sen}^2\theta + \cos^2\theta + r^2}$$

$$\sqrt{1+r^2}$$

$$\int_0^{2\pi} \int_0^{1} \sqrt{r^2+1}\, dr\, d\theta$$

$$\tan\theta = \frac{r}{1}$$

$$r = \tan\theta$$



$$\int_0^1 \sqrt{\tan^2 + 1}\; dr$$

$$\int_0^1 \sec\, dr$$

$$\ln|\sec\theta + \tan\theta|$$

$$\ln|\sqrt{r^2+1} + r| \Big|_0^1$$

$2\pi\left(\ln|\sqrt{2}+1|\right)$

b) Suponga que la superficie tiene una densidad de $\sqrt{x^2+y^2+1}$

$$\sqrt{(r\cos\theta)^2+(r\,\text{sen}\,\theta)^2+1}$$

$$\sqrt{r^2+1}$$

$$\int_0^{2\pi}\int_0^1 \sqrt{r^2+1}\,\sqrt{r^2+1} \quad\Big|\quad 2\pi\left[\frac{4}{3}\right]$$

$$\int_0^{2\pi}\int_0^1 r^2+1\,dr\,d\theta \quad = \quad \boxed{\frac{8\pi}{3}}$$

$$\frac{r^3}{3}+r\Big|_0^1 =$$

$$2\pi\left[\frac{1}{3}+\frac{1}{1}\right] =$$

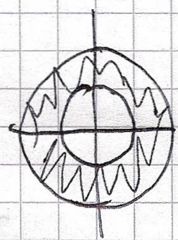
28 Halla el flujo del campo $\vec{F}$

$\vec{F}(x,y,z) = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$ a traves de S

$x^2+y^2+z^2=4$

$1 \leq x^2+y^2 \leq 4$

$x = 2\,\text{sen}\,\phi \cos\theta$
$y = 2\,\text{sen}\,\phi\,\text{sen}\,\theta$
$z = 2\cos\phi$



$1+z^2=4$

$z^2=3$

$z=\sqrt{3}$

$\cos\phi = \frac{\sqrt{3}}{2}$

$\phi = \frac{\pi}{6}$

$\cancel{1+z^2=4}$

$\cancel{z^2=0}$

$\cancel{z=0}$

$\cancel{\cos\phi=0}$

$r_\phi = \begin{pmatrix} 2\cos\phi\cos\theta \\ 2\cos\phi\,\text{sen}\,\theta \\ -2\,\text{sen}\,\phi \end{pmatrix}$

$r_\theta = \begin{pmatrix} -2\,\text{sen}\,\phi\,\text{sen}\,\theta \\ 2\,\text{sen}\,\phi\cos\theta \\ 0 \end{pmatrix}$

$\frac{\pi}{6} \leq \phi \leq \frac{\pi}{2}$

$0 \leq \theta \leq 2\pi$

$$\begin{vmatrix} i & j & k \\ 2\cos\phi\cos\theta & 2\cos\phi\,\text{sen}\,\theta & -2\,\text{sen}\,\phi \\ -2\,\text{sen}\,\phi\,\text{sen}\,\theta & 2\,\text{sen}\,\phi\cos\theta & 0 \end{vmatrix}$$

$4\,\text{sen}^2\phi\cos\theta$

$-(0-4\,\text{sen}^2\phi\,\text{sen}\,\theta)$

$4\,\text{sen}\,\phi\cos\phi\cos^2\theta + 4\,\text{sen}\,\phi\cos\phi\,\text{sen}^2\theta$

$$\begin{pmatrix} 4\,\text{sen}^2\phi\,\cos\theta \\ 4\,\text{sen}^2\phi\,\text{sen}\,\theta \\ 4\,\text{sen}\,\phi \cdot \cos\phi \end{pmatrix} = r_\phi \times r_\phi$$

ϕ

$\phi_1 = \frac{\pi}{6}$

$\phi_2 = \pi - \frac{\pi}{6}$

$\phi_2 = \frac{5\pi}{6}$

$\frac{\pi}{6} \leq \phi \leq \frac{5\pi}{6}$

$$\iint \begin{pmatrix} 2\,\text{sen}\,\phi\,\cos\theta \\ 2\,\text{sen}\,\phi\,\text{sen}\,\theta \\ 1 \end{pmatrix} \begin{pmatrix} 4\,\text{sen}^2\phi\,\cos\theta \\ 4\,\text{sen}^2\phi\,\text{sen}\,\theta \\ 4\,\text{sen}\,\phi\,\cos\phi \end{pmatrix}$$

$$8\,\text{sen}^3\phi\,\cos^2\theta + 8\,\text{sen}^3\phi\,\text{sen}^2\theta + 4\,\text{sen}\,\phi\,\cos\phi$$

$$\int_0^{2\pi}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 8\,\text{sen}^3\phi + 4\,\text{sen}\,\phi\,\cos\phi$$

$u = \text{sen}\,\phi$

$du = \cos\phi$

$$\frac{8}{2}\int_0^{2\pi}\int_{\pi/6}^{5\pi/6} \frac{(1-\cos(2\phi))}{1}\,\text{sen}(\phi) + 4\int_0^{2\pi}\int_{\pi/6 \to \frac{1}{2}}^{5\pi/6 \to \frac{1}{2}} U\,dU$$

$$4\int_0^{2\pi}\int_{\pi/6}^{5\pi/6} \text{sen}(\phi) - \text{sen}\,\phi\,(\cos(2\phi)) \quad + 8\pi \cancel{\left[\frac{U^2}{2}\Big|_{1/2}^{1/2}\right]} \to 0$$

$$8\pi \int_{\pi/6}^{5\pi/6} \text{sen}\,\phi \;-\; 8\pi \int_{\pi/6}^{5\pi/6} \text{sen}\,\phi \cdot \cos(2\phi)\;d\phi$$

$$8\pi \quad -\cos\left(\frac{5\pi}{6}\right) + \cos\left(\frac{\pi}{6}\right) \quad - \frac{1}{2}\left[0 - \frac{\sqrt{3}}{2} - \left(\cancel{0} + \frac{\sqrt{3}}{2}\right)\right]$$

$$\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \qquad\qquad -\sqrt{3}$$

$$8\pi\sqrt{3} \qquad - \frac{8\pi\sqrt{3}}{2}$$

$$8\pi\sqrt{3} - 4\pi\sqrt{3}$$

$$\boxed{4\pi\sqrt{3}}$$

29 Considere el campo magnetico definido por

$$\vec{F}(x,y,z) = \begin{pmatrix} x \\ -y \\ 1 \end{pmatrix} \qquad \text{sobre } S$$

$$x^2+y^2+z^2=1$$

$$x^2+y^2 \leq 4$$

$$z \geq 0$$

$$z^2 = x^2 + y^2 - 1 \qquad x = x$$

$$z = \sqrt{x^2+y^2-1} \qquad y = y$$

$$z = \sqrt{x^2+y^2-1}$$

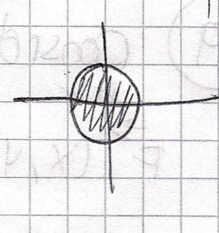
$r_x \times r_y$

$$r_x = \begin{pmatrix} 1 \\ 0 \\ \frac{x}{\sqrt{x^2+y^2-1}} \end{pmatrix}$$

$$r_y = \begin{pmatrix} 0 \\ 1 \\ \frac{y}{\sqrt{x^2+y^2-1}} \end{pmatrix}$$

$$\begin{vmatrix} i & j & k \\ 1 & 0 & \frac{x}{\sqrt{x^2+y^2-1}} \\ 0 & 1 & \frac{y}{\sqrt{x^2+y^2-1}} \end{vmatrix} = \begin{pmatrix} \frac{x}{\sqrt{x^2+y^2-1}} \\ \frac{-y}{\sqrt{x^2+y^2-1}} \\ 1 \end{pmatrix}$$

$$x^2 + y^2 \le 4$$



$$\frac{x^2}{\sqrt{x^2+y^2-1}} + \frac{y^2}{\sqrt{x^2+y^2-1}} + 1$$

Polares